СПбГУТ)))

Single Frequency Networks
Practice 0: Fourier Series

Department of Television and Metrology
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Targets

In this practice we are going to work with the Fourier series in MATLAB, we retake the example of a periodic square signal to represent it. The main targets of this practice is to familiarize with MATLAB command used to plot signals, generating the appropriate frequency vectors for its representation and review the concept of the Fourier series.

Description

From the theory lessons we know the expression for a periodic square signal, mathematically in one period:

$x(t)$

$$x(t)$$

$-T_0/2$

$T_0/2$

t

This signal can be represented in time also as the Fourier series indicates:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{T_0}kt}$$

Where c_k can be obtained by applying:

$$c_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j\frac{2\pi}{T_0}kt}dt$$

In this case we have already worked with this signal and we already know the expression for the differentc_k:

$$c_k = \begin{cases} \frac{2}{j\pi k}, \; k = \pm 1, \pm 3, \pm 5, \dots \\ 0, \; k = 0, \pm 2, \pm 4, \dots \end{cases}$$

Step 1

Using the definition for $x(t)$ that the Fourier series gives us, we are going to form a “square periodic signal” with a determinate number of harmonics. We are going to represent 4 periods of this signal and see the difference in shape with different number of harmonics.
We are going to program a function that receives as an input the number of harmonics to use to build the signal and its period.
To define the number of periods we will use a time vector defined as follows:

```
t=[-2:tStep:2]*Period; % Time vector definition
```

This will give us 4 periods of our signal with a sample time of `tStep*Period`.
After this we will initialize to zero our signal vector and our coefficient vector:

```
xt=zeros(size(t));
cK=zeros(1,NumberHarmonics);
```

After that we will need to proceed with a for loop to accumulate in xt the different harmonics. Note that because of being $x(t)$ a real signal the different harmonics can be grouped in sines ($c_k^* = c_{-k}$), and we need to multiply the original expression of c_k by two. After, we will plot the resulting vector and try different number of harmonics.

Step 2

In this step we are going to plot the Fourier coefficients to see the spectrum of the signal. In order to achieve so, we will need to create the appropriate frequency vector and coefficients vector. Take into account that to get a non-normalized frequency vector we need to divide by the period of the square signal.

Step 3

We are going to perform the FFT of the generated xt signal to check that it has the properties it should (number of harmonics).

Key functions in Matlab

- fft(x): is the Discrete Fourier transform (DFT) of vector x
- sin(x): is the sine of the elements of x
- zeros(M,N): is an M-by-N matrix of zeros
- plot(x,y): plots vector y versus vector x
- stem(x,y): plots the data sequence y at the values specified in x