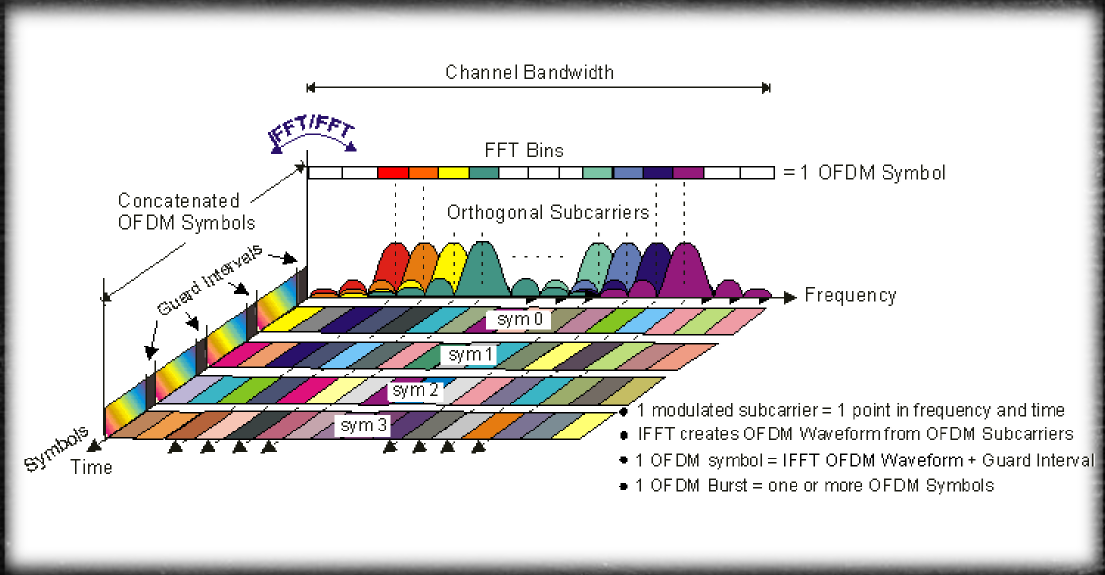
Introduction to OFDM

Basic concepts

Definitions

- Orthogonal Frequency Division Multiplexion
  - A number of orthogonal carriers bear the information
  - Resilient to time synchronization errors
  - Very high spectral efficiency



- OFDM is a wide used transmission technique
  - ADSL
  - IEEE 802.11a/g/n and WiMAX
  - DVB-T , DVB-H , DVB-T2, DVB-NGH.

- The energy of a signal is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

- There are signals with $E = \infty$, as periodic signals for them it is defined the average power:

$$P = \lim_{T_0 \to \infty} \frac{1}{2T_0} \int_{-T_0}^{T_0} |x(t)|^2 \, dt$$

- Signals can be classified in
  - Energy signals: $0 < E < \infty$
  - Power signals: $0 < P < \infty$

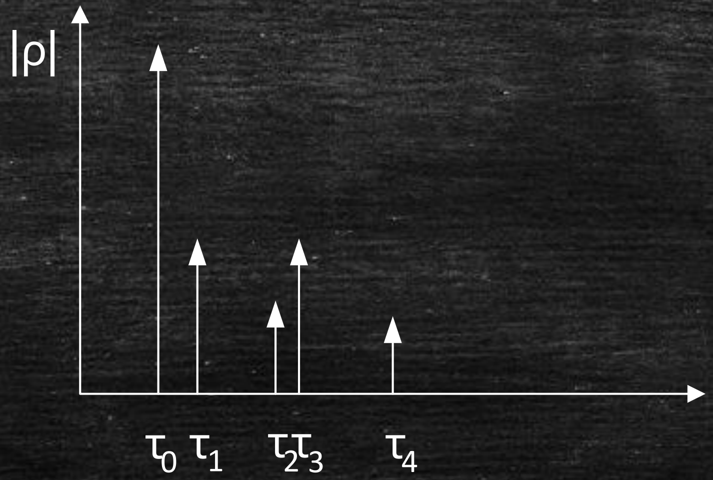
Propagation scenarios

- In wireless communications there are much harder propagation conditions than in wired communications
- There are different propagations paths from the transmitter to the receiver (multi path propagation)

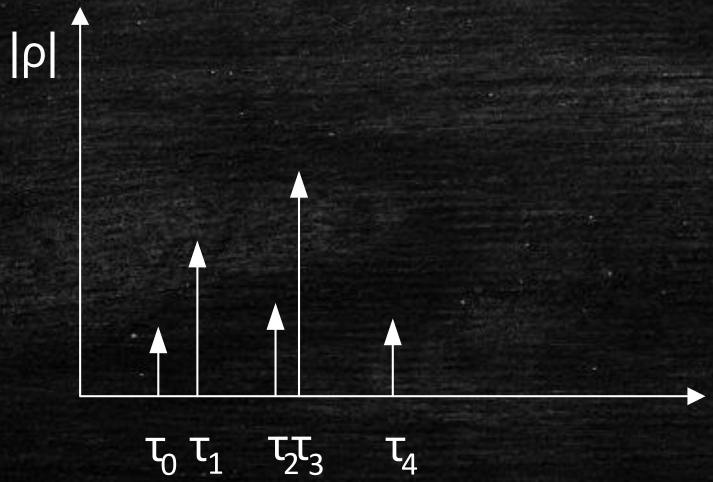
- The propagation scenarios are usually modelled as a FIR filter which taps represent:
  - Delay (τ)
  - Path complex gain (ρ)

- The propagation scenarios can be divided in two main types
  - Ricean (there is line of sight and hence direct ray)
  - Rayleigh (there is no line of sight and hence no direct ray)

- The Ricean channel looks like:

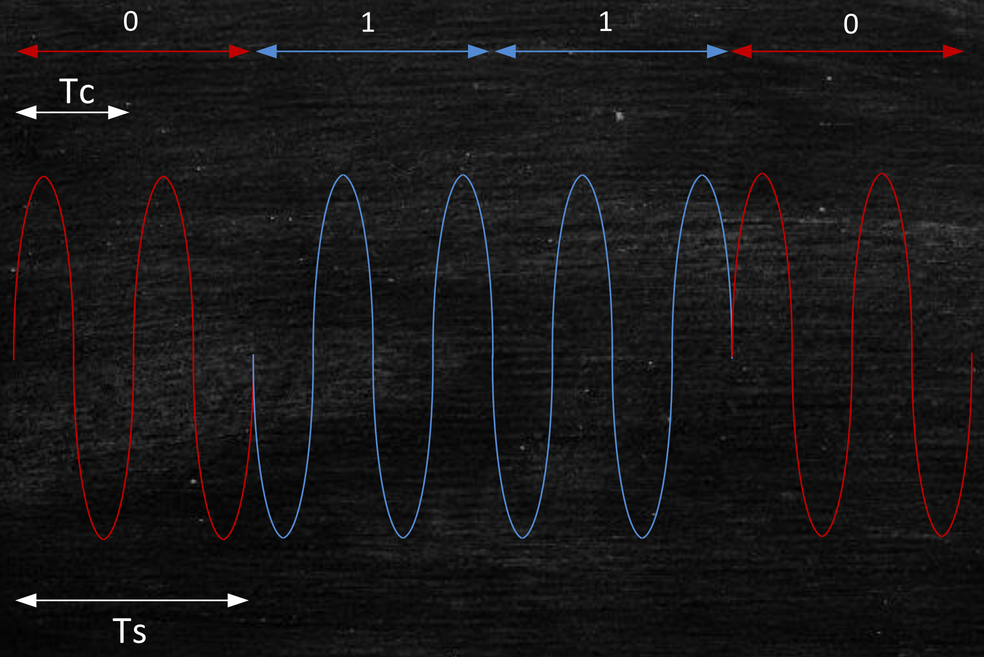


- The Rayleigh channel looks like:

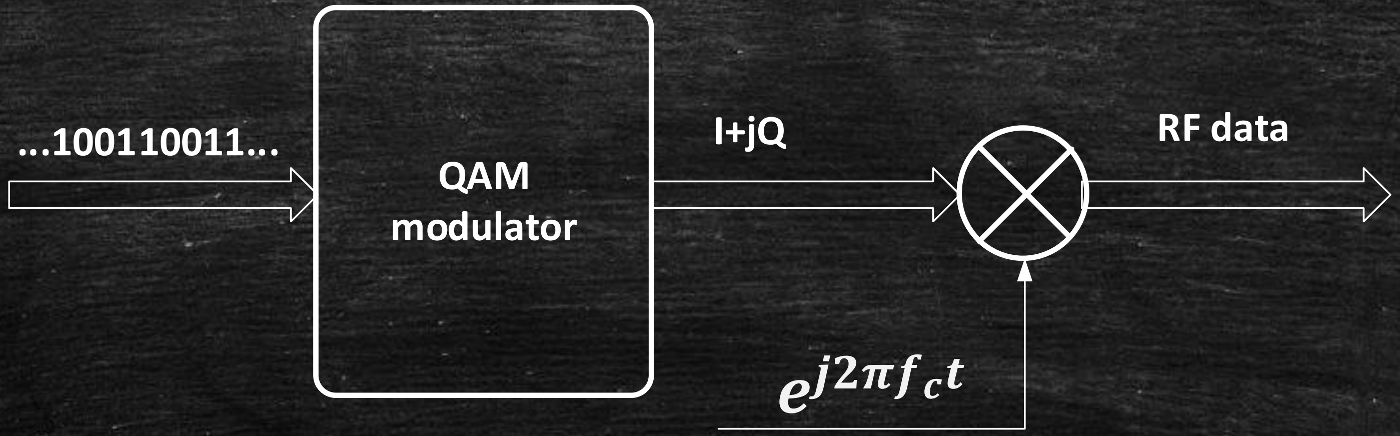


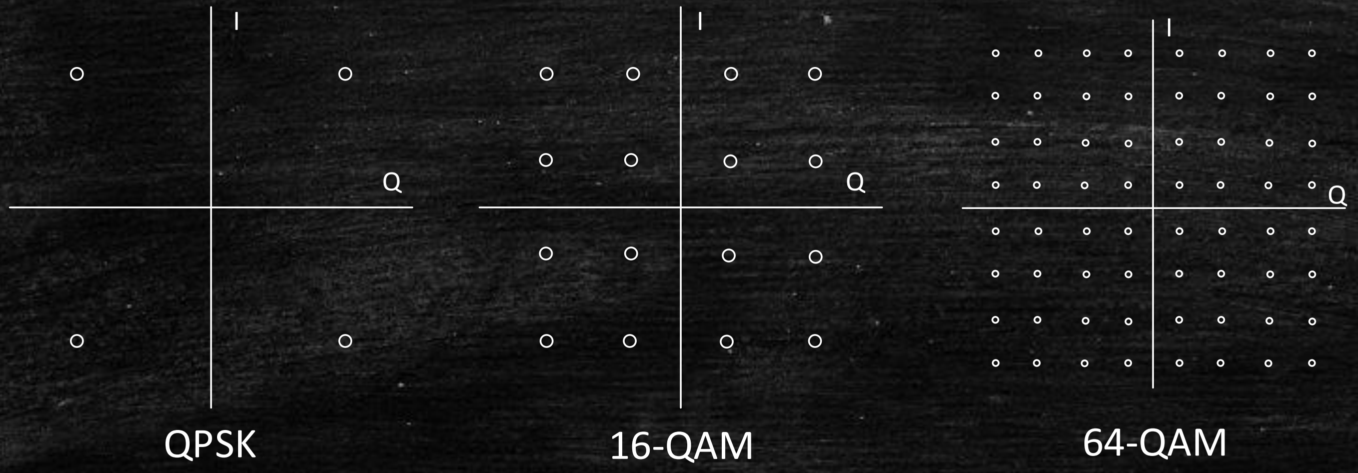
Single carrier modulation

- Lets suppose a single carrier BPSK modulation
  - Carrier frequency, $f_c = 1/T_c$; 1 bit per symbol
    - $b_i = 1$, 180º degrees phase shift
    - $b_i = 0$, 0º degrees phase shift
    - Bit rate $R = 1/T_s$, where T_s represents the symbol time

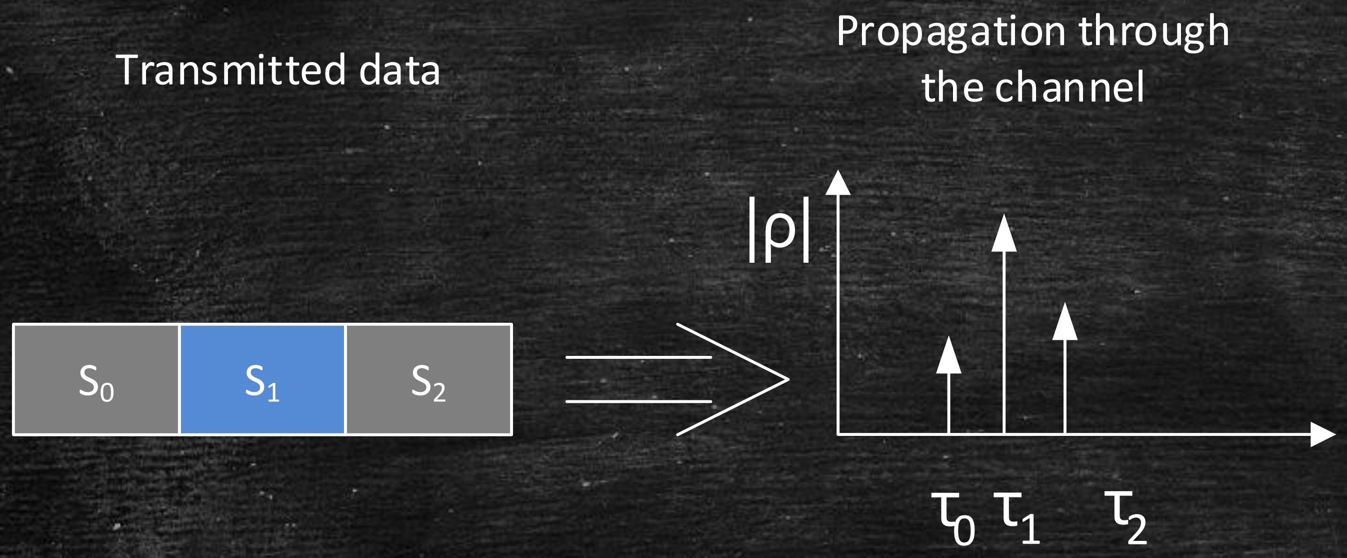


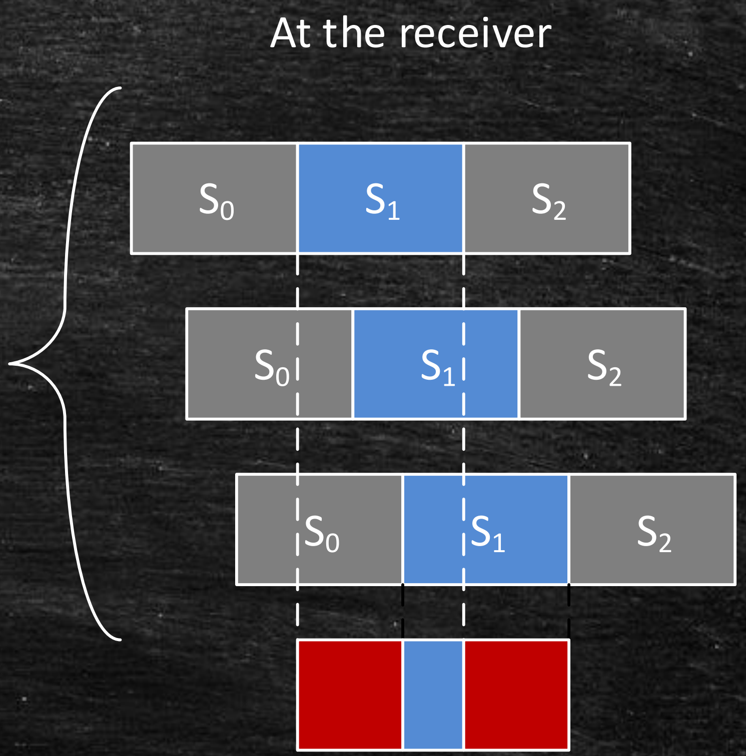
- A simple transmitter example





- Different echoes effect:





  – The red coloured part is affected by ISI
    - Interference produced by other symbols
  – The symbol at the receiver seems longer in time
  – In the following part we will focus on the blue part
    - Only interference from the own symbol delayed

- All the stated is observed in the time domain, but what is its interpretation in the frequency domain?

$$S(t) \overset{FFT}{\longleftrightarrow} S(f)$$

$$S(t-\tau) \overset{FFT}{\longleftrightarrow} e^{-j2\pi f\tau} S(f)$$

- For a channel as the one shown in the previous example:

$$S_{Rx}(f) = \rho_0 S(f) e^{-j2\pi f\tau_0} + \rho_1 S(f) e^{-j2\pi f\tau_1} + \rho_2 S(f) e^{-j2\pi f\tau_2}$$

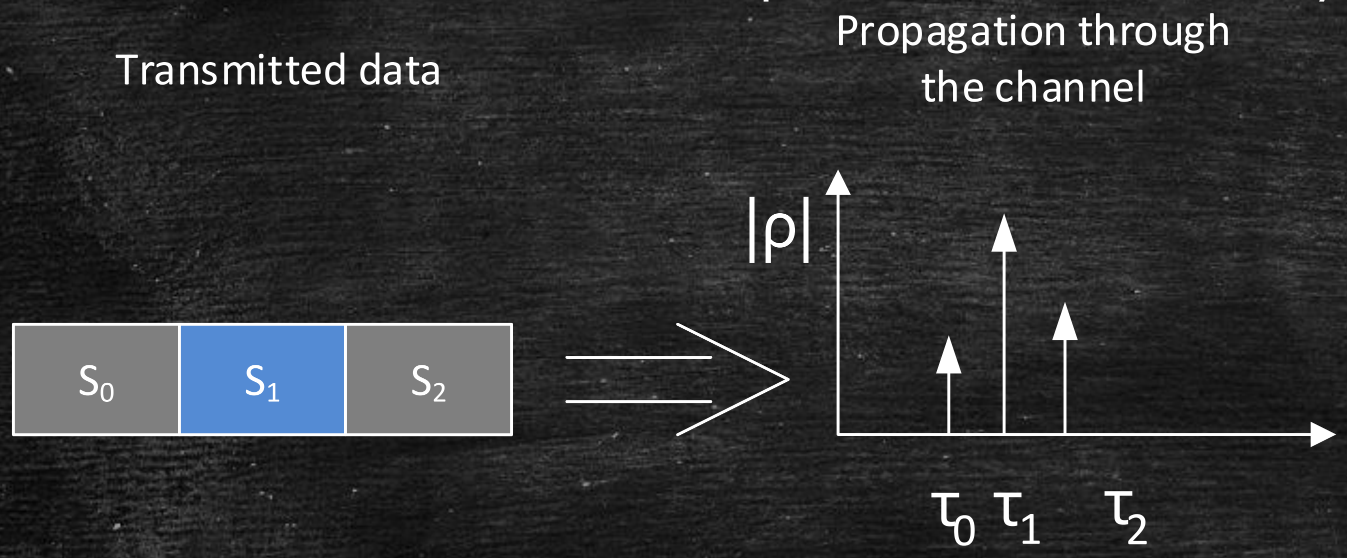
$$S_{Rx}(f) = S(f)\left(\rho_0 e^{-j2\pi f\tau_0} + \rho_1 e^{-j2\pi f\tau_1} + \rho_2 e^{-j2\pi f\tau_2}\right)$$

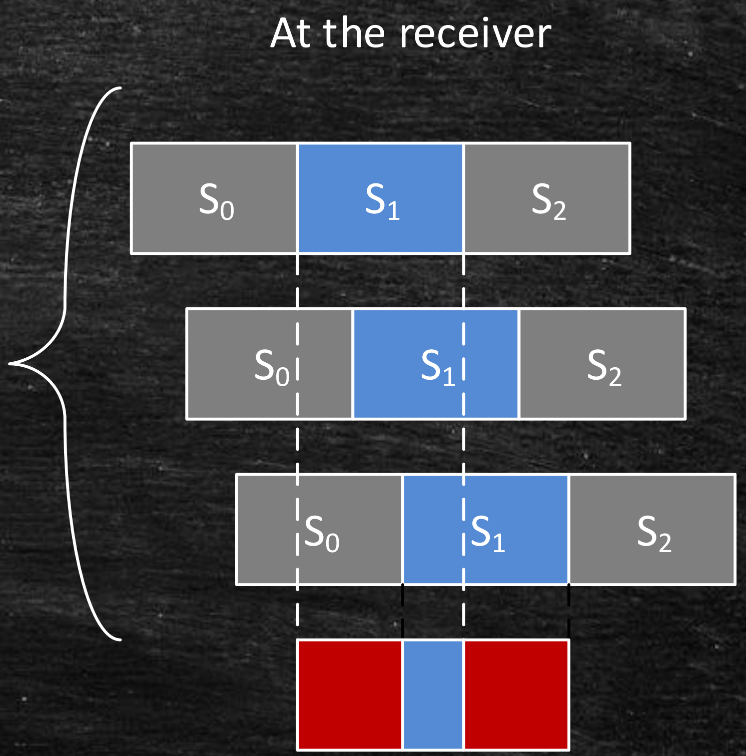
$$S_{Rx}(f) = S(f)H(f)$$

- $H(f)$ represents the channel response in the frequency domain

- The channel effect can produce high performance loss
  - The combination of the different paths can be constructive or destructive
  - The destructive combinations can lead to huge attenuation (fadings) even to erasure events
  - The channel equalization is necessary to solve the problems associated to the channel fadings and erasures

- Now we will focus on the red part of the received symbol

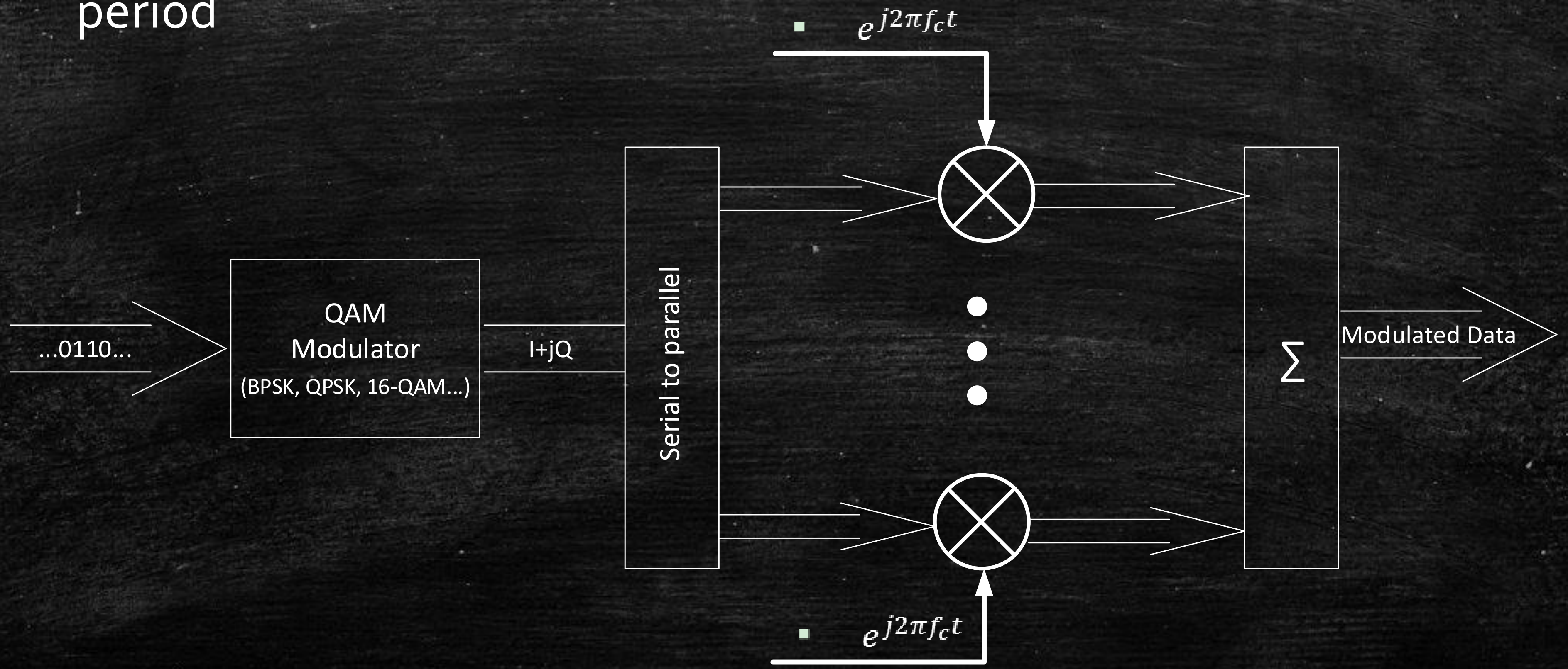




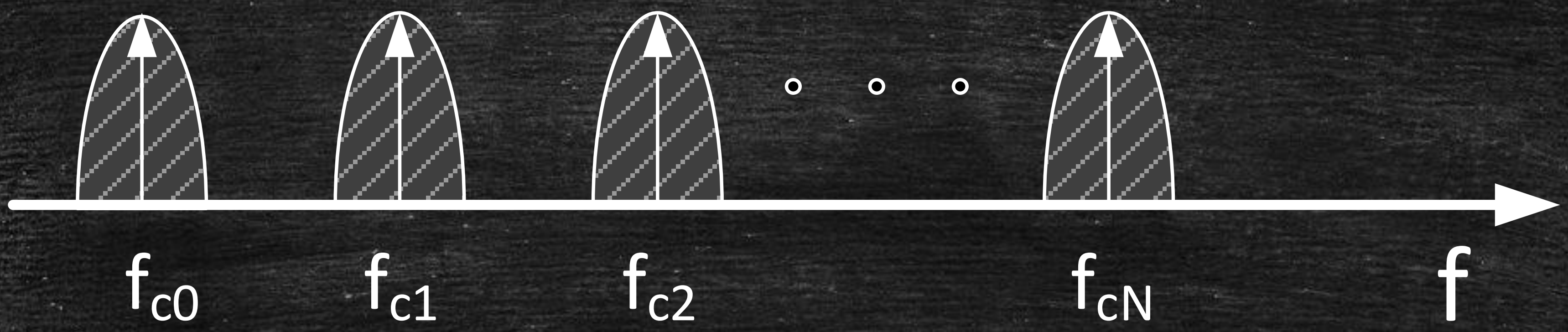
  - Sum of different delayed symbols
  - In order to recover the information the symbol
    - T_s must be much longer than the difference between the first ray and the last one of the channel (τ_{max})
    - The longer the symbol is the less rate we obtain

Multi carrier modulation

- The solution for the aforementioned problem is parallelize
  - Send the information through different carriers with higher symbol period



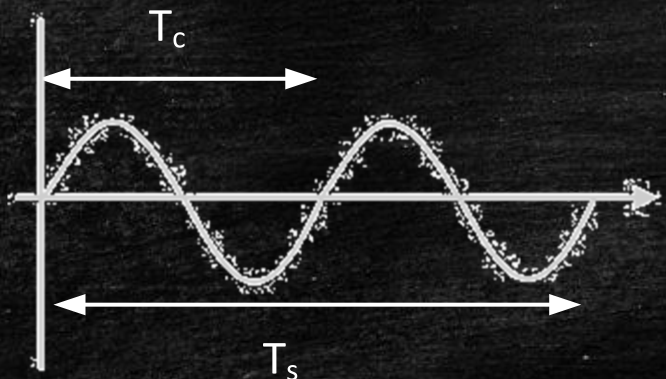
- The spectrum at the output of the modulator:



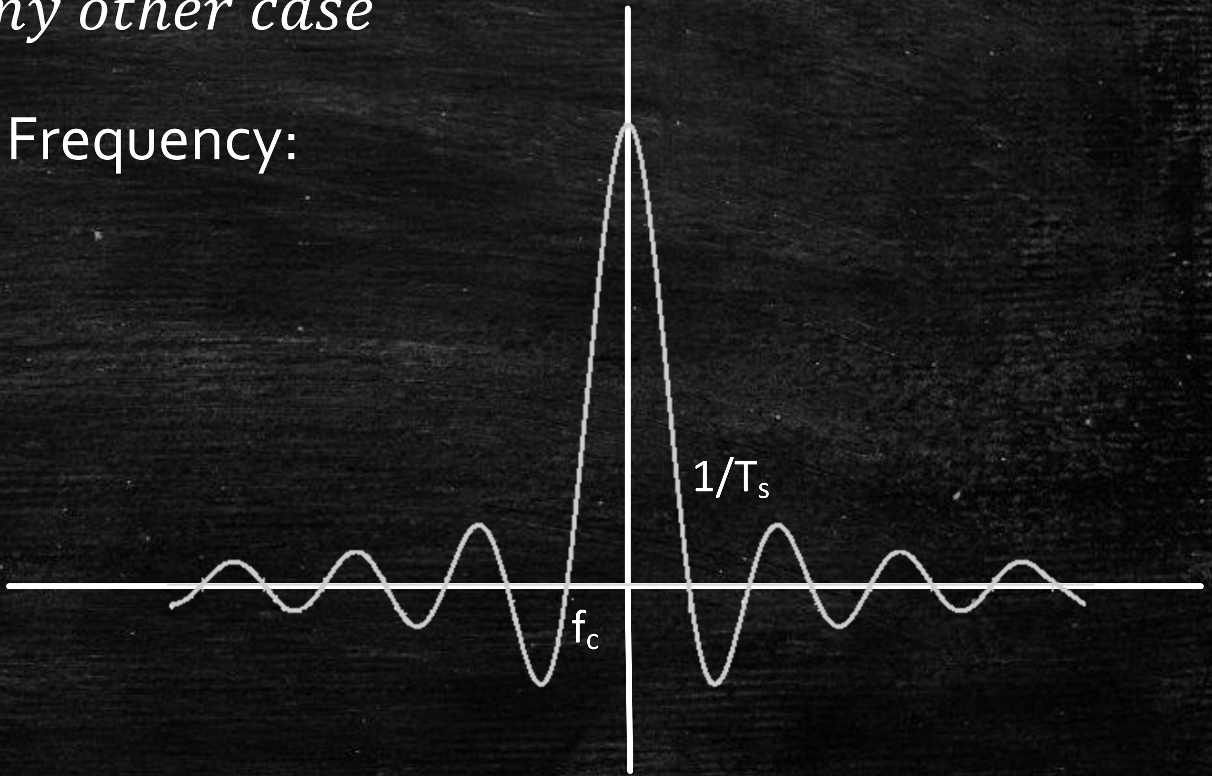
- For a N+1 carriers, input data rate R, and BPSK modulation
  - For a same data rate R, the symbol time of a multi-carrier is N+1 times higher than for a single carrier
  - For a given τ_{max} the data rate is higher than in a single carrier
  - The spectrum occupied is much wider!

Orthogonal Frequency Division Multiplexion

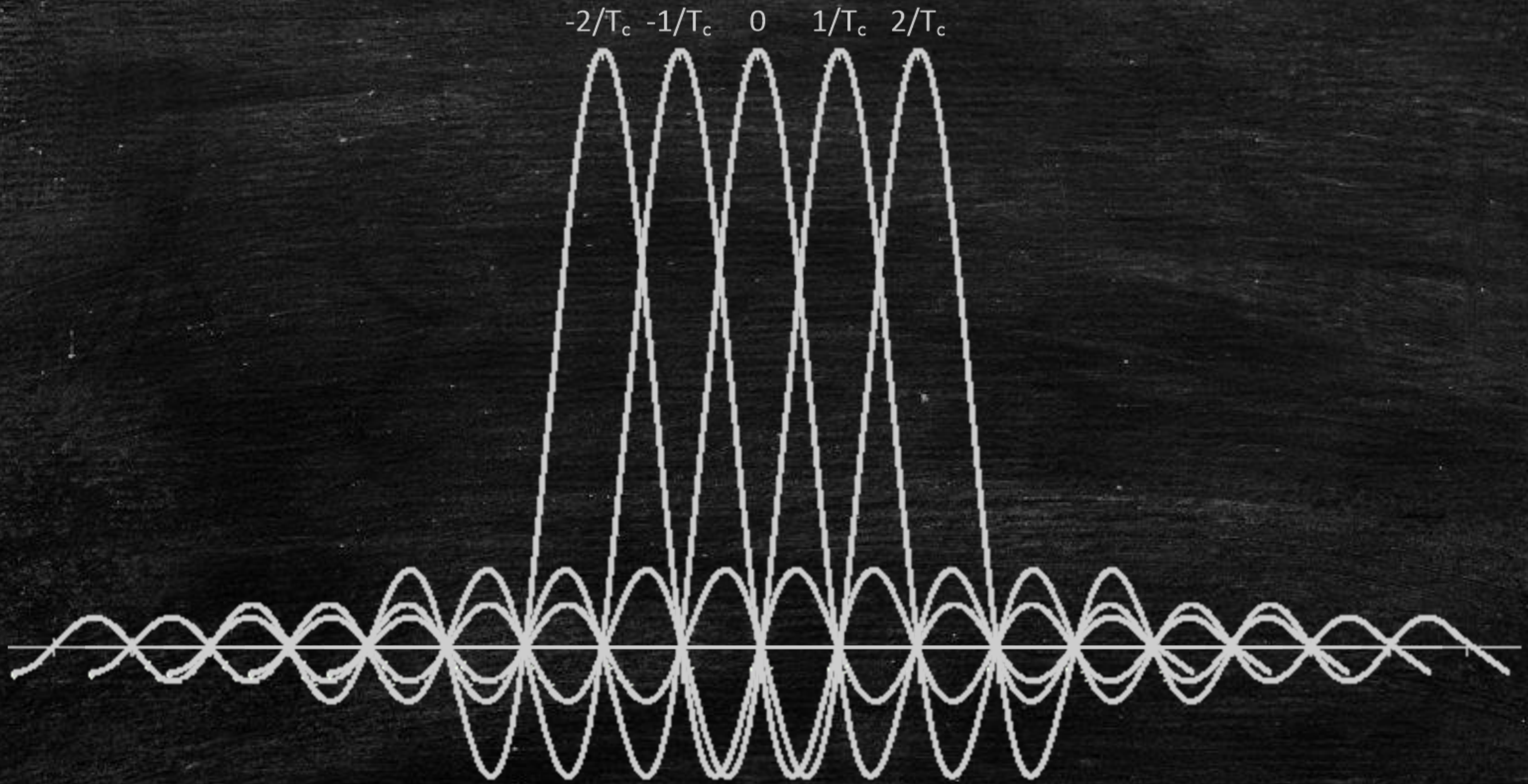
- Spectral characteristics of a FDM signal:
- $\phi_c = e^{j2\pi f_c t} rect\left(\frac{t}{T_s}\right) = \begin{cases} e^{j2\pi f_c t} \; if \; 0 \le t \le T_s \\ 0 \; in \; any \; other \; case \end{cases}$
- Time (real part):



Frequency:



- If the N carriers in a OFDM symbol satisfy:
  - $f_{ck} = k/T_c \; with \; k = -N/2, \dots, 0, \dots, N/2 - 1$
- The carriers are orthogonal
  - The maximum of a carrier coincides with the zeros of the others



- Mathematically the expression for a determinate symbol:

$$s(t) = \sum_{k=0}^{N-1} a_k \, e^{j2\pi f_{ck} t} rect\left(\frac{t}{T_s}\right)$$

  - a_k represents the k-th output symbol of the QAM modulator ($a_k = I_k + jQ_k$)
  - N represents the number of carriers in the OFDM symbol
  - $f_{ck} = k/T_c \; with \; k = 0,1, \ldots, N-1$

- The problem is now obtaining that many analogue oscillators!

- In the digital domain, if sampled with *N* samples, a symbol:

$$T_{samp} = T_s/N$$

- The time becomes discrete, $t = nT_{samp}$, and then:

$$f_{ck}t = \left(\frac{k}{T_S}\right) n(T_S/N) = kn/N$$

$$s[n] = \sum_{k=0}^{N-1} a_k \, e^{\frac{j2\pi kn}{N}} rect\left[\frac{n - \frac{N}{2}}{N}\right]$$

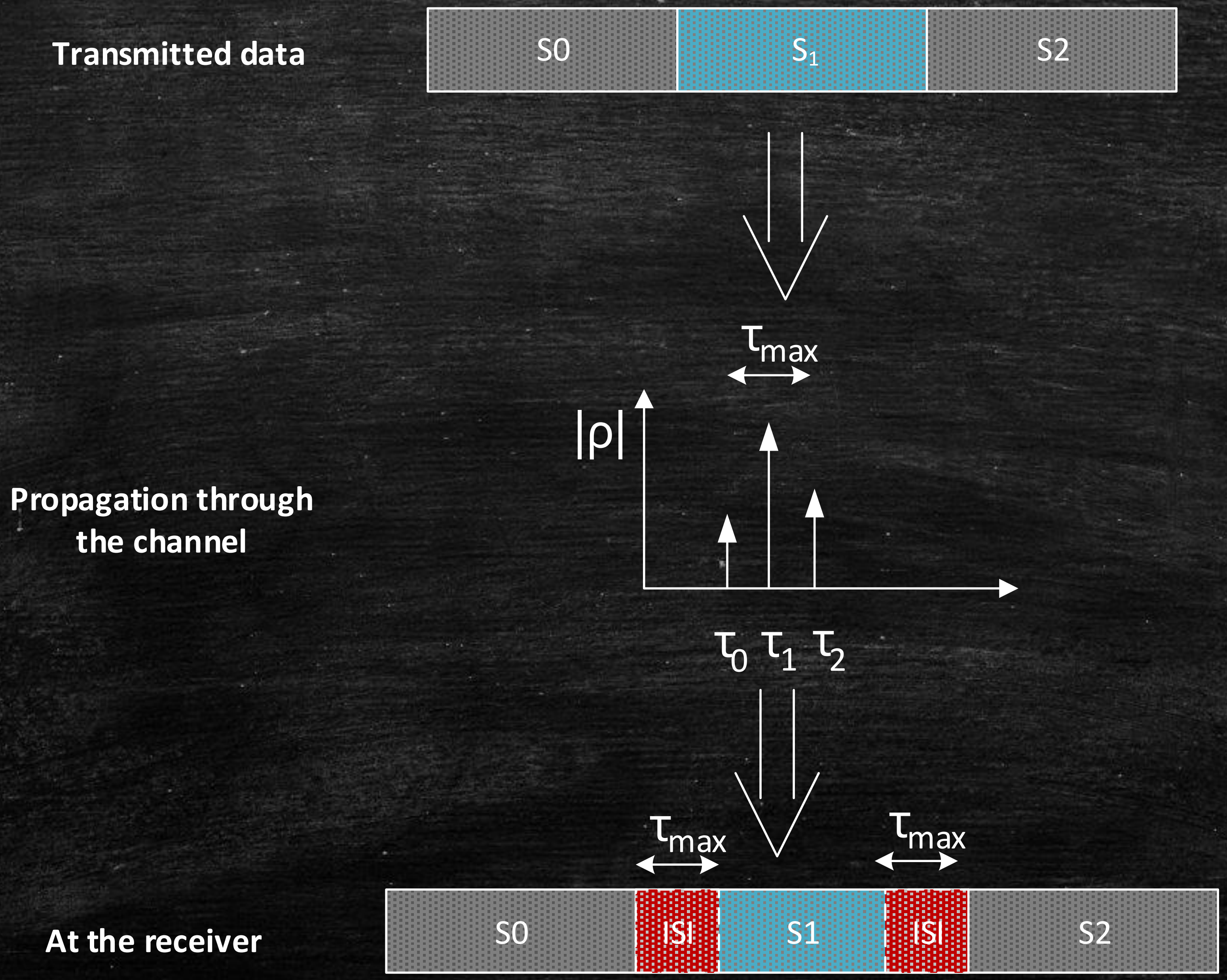
$$s[n] = \sum_{k=0}^{N-1} a_k \, e^{j\frac{2\pi}{N}kn} \quad n \in [0, N-1]$$

- Looking carefully to the expression achieved before:
  - It matches with the IDFT of a_k!

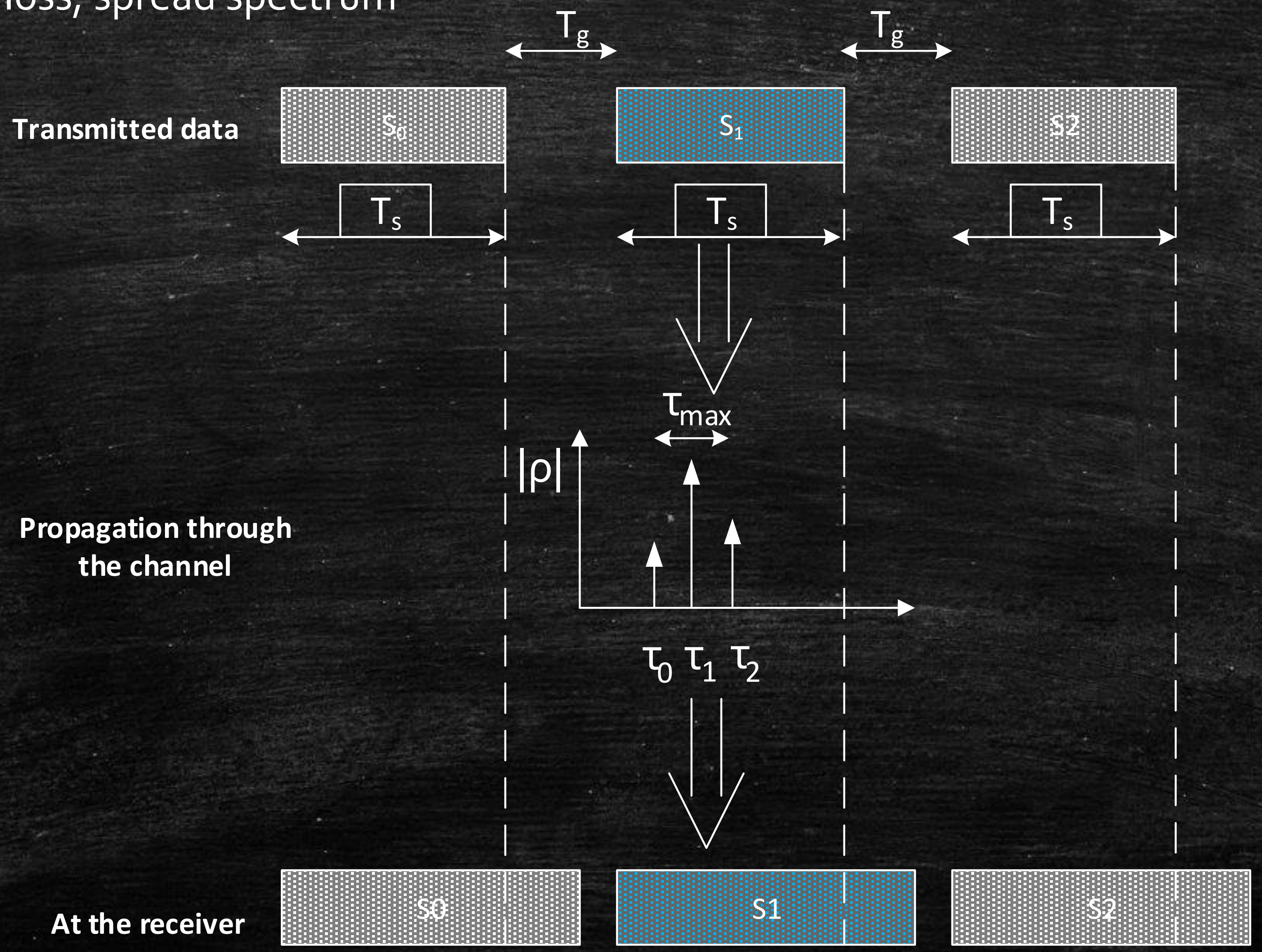
$$s[n] = \sum_{k=0}^{N-1} a_k \, e^{j\frac{2\pi}{N}kn} = IDFT[a_k]$$

  - Every a_k represents the amplitude and phase of the carriers
  - We are coming form the frequency domain to the time domain
    - Every block previous to the IDFT in an OFDM system belongs to the frequency domain
    - Every block after the IDFT in an OFDM system belongs to the time domain

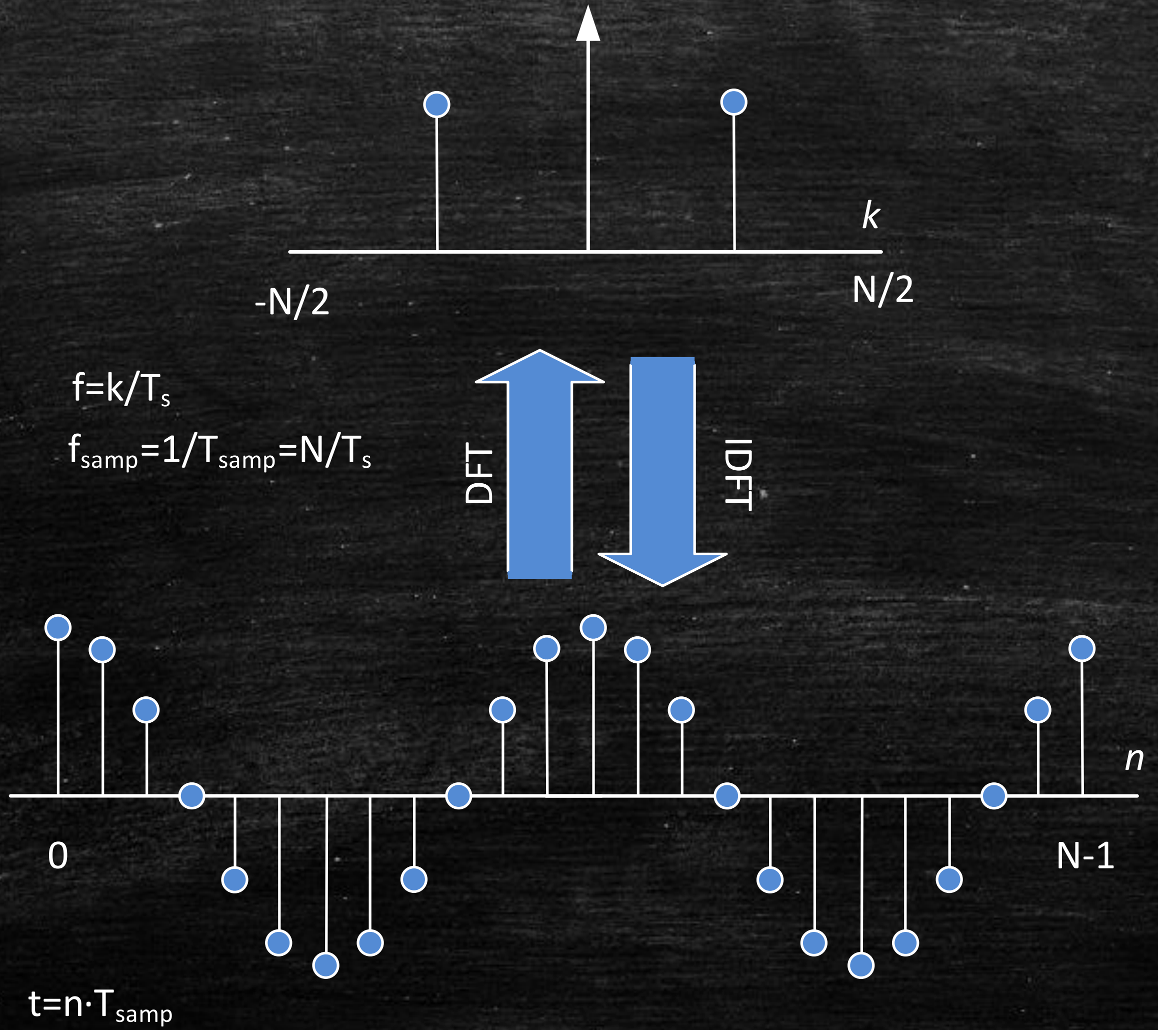
- There is still a problem to be solved: ISI



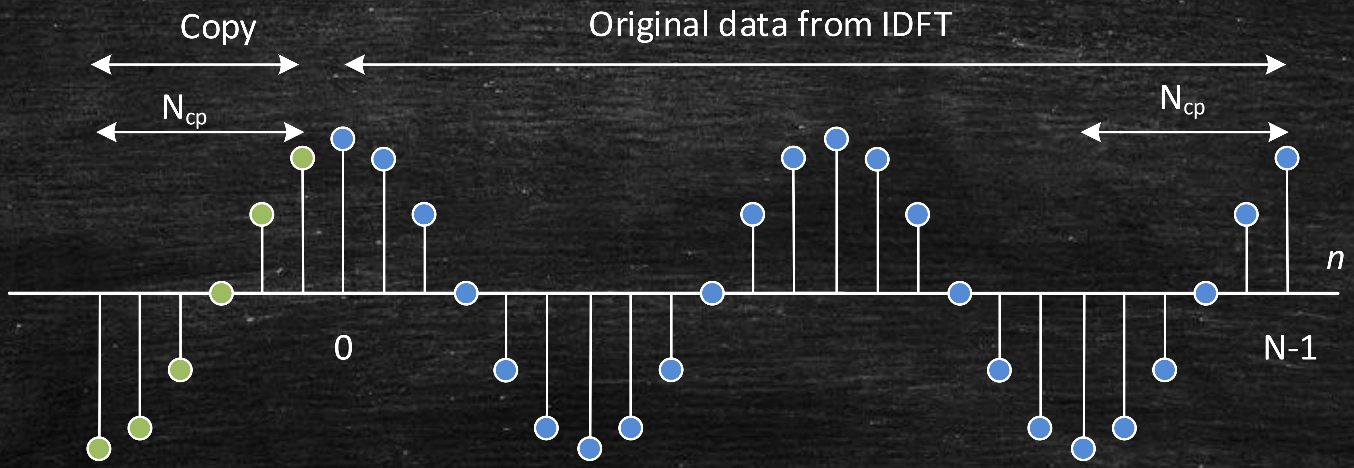
- With a longer symbol time the ISI is reduced (in %) but the problem still exists
  - A possible solution is to insert a guard period between symbols
    - Capacity loss, spread spectrum



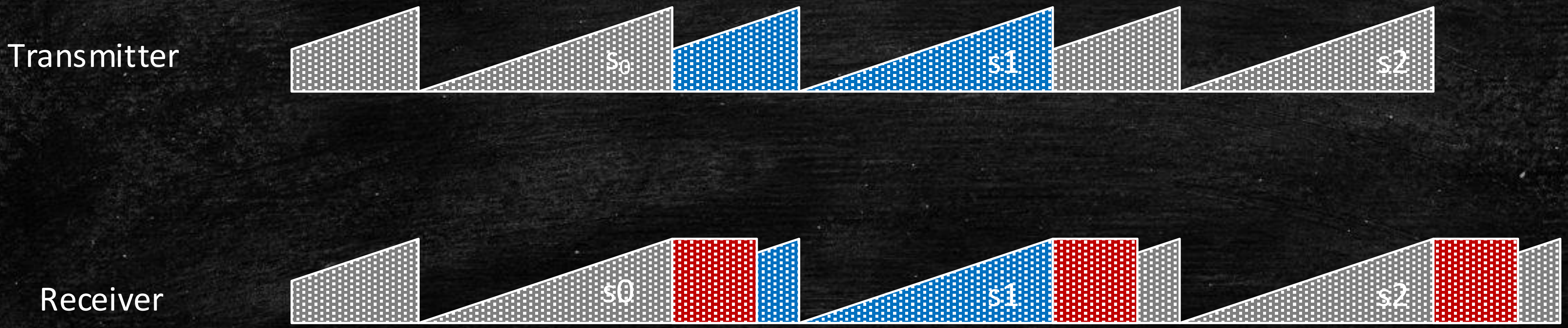
- Another way to cope with ISI is the cyclic prefix
  - DFT and IDFT are applied to periodic signals
  - The result of the IDFT is a period of a periodic signal in time



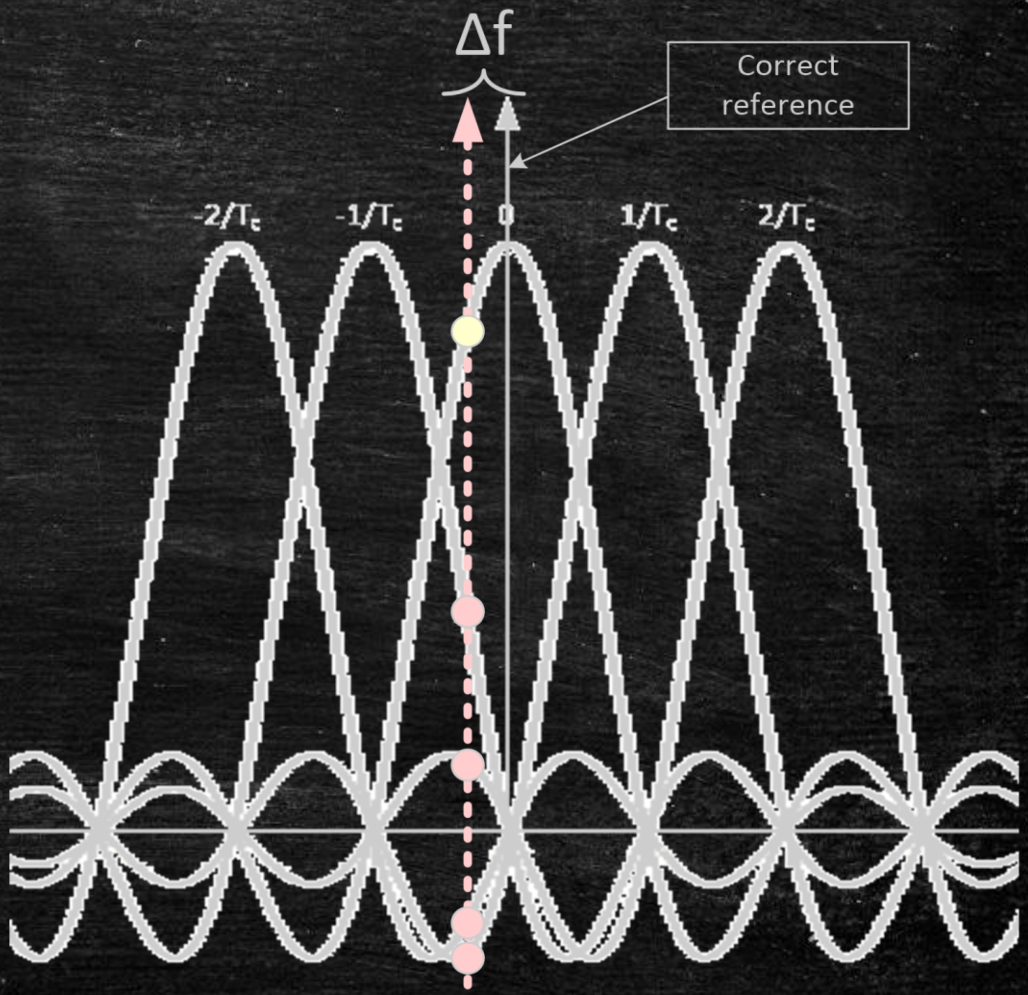
- The cyclic prefix consists in taking the last N_{cp} samples of the signal and copy it at the beginning of the symbol at the receiver these samples are discarded



  – Reduces the capacity but doesn't spread the spectrum



- There is an additional problem with OFDM
  - The frequency reference in both transmitter and receiver are not usually exactly same (oscillators deviations, mismatching of the elements …)
  - This leads to a orthogonality loss because the sampling instant is not exactly in the maximum (and cero of the other carriers)
  - This is additional interference coming from other carriers (ICI)



THANKS !

Any questions?

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