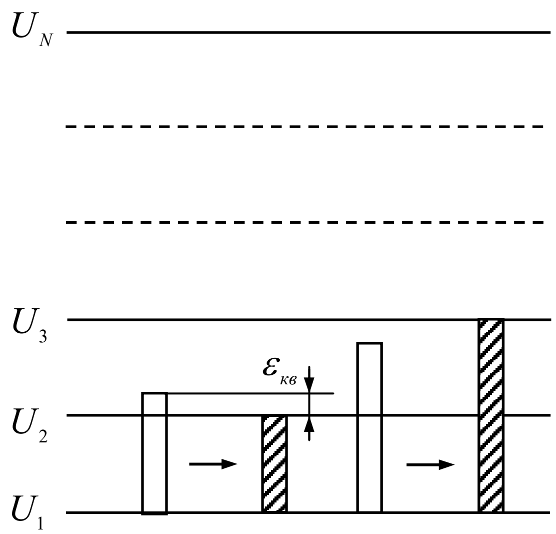
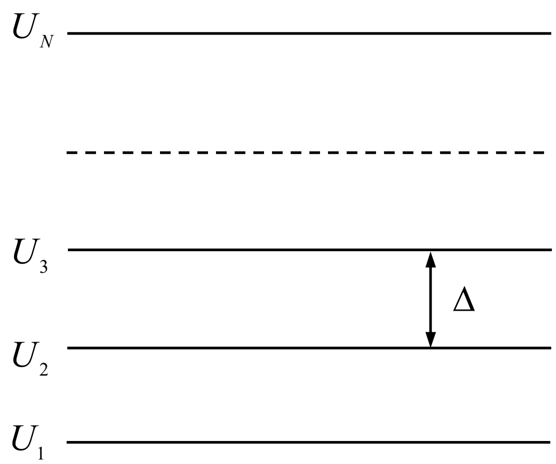
Квантование, кодирование (основные понятия)



$$\underbrace{1\ldots\ldots1}_{m}$$

$$N = 2^m$$

$$\underbrace{0\ldots\ldots0}_{m}$$

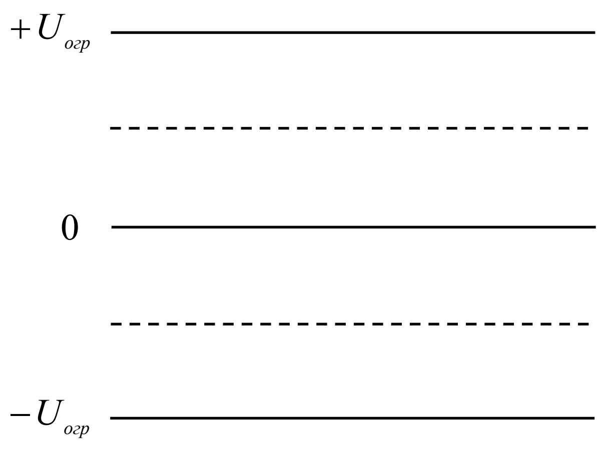


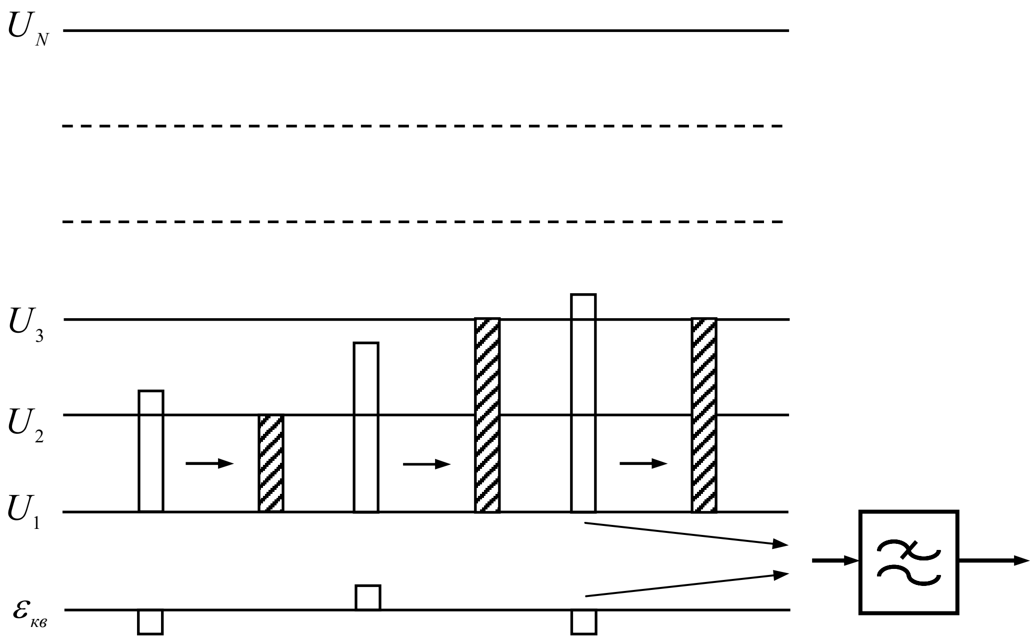
$\Delta = const$ р. шк.

$$\varepsilon_{кв.\max} = \frac{\Delta}{2}$$

$\Delta \neq const$ н. шк.

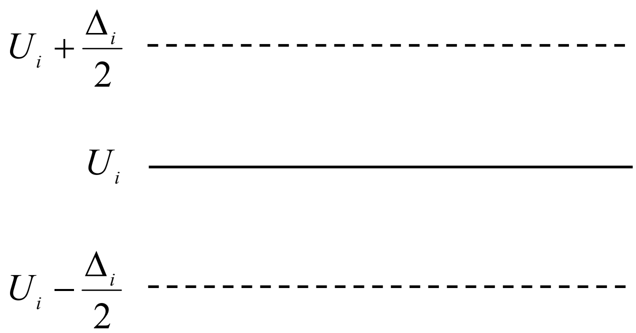
$$\varepsilon_{кв.i\max} = \frac{\Delta_i}{2}$$





$$A_{з\ кв} = 10\lg \frac{P_c}{\sigma^2_{кв}}$$

Мощность шума квантования



$$\sigma^2_{кв\ i} = \int_{U_i - \frac{\Delta_i}{2}}^{U_i + \frac{\Delta_i}{2}} w(U)(U - U_i)^2\, dU$$

$$w(U)\Big|_{U_i - \frac{\Delta_i}{2}}^{U_i + \frac{\Delta_i}{2}} = const = w(U_i)$$

$$\sigma^2_{кв\,i} = w(U_i)\int\limits_{U_i-\frac{\Delta_i}{2}}^{U_i+\frac{\Delta_i}{2}}(U-U_i)^2\,dU = w(U_i)\left.\frac{(U-U_i)^3}{3}\right|_{U_i-\frac{\Delta_i}{2}}^{U_i+\frac{\Delta_i}{2}} = w(U_i)\frac{\Delta_i^3}{12}$$

$$p_i = \int\limits_{U_i-\frac{\Delta_i}{2}}^{U_i+\frac{\Delta_i}{2}} w(U)\,dU = w(U_i)\int\limits_{U_i-\frac{\Delta_i}{2}}^{U_i+\frac{\Delta_i}{2}} dU = w(U_i)\,U\Big|_{U_i-\frac{\Delta_i}{2}}^{U_i+\frac{\Delta_i}{2}} = w(U_i)\Delta_i$$

$$\sigma^2_{кв\,i} = p_i\frac{\Delta_i^2}{12} \qquad \sigma^2_{кв} = \sum_{i=1}^{N}\sigma^2_{кв\,i} = \sum_{i=1}^{N} p_i\frac{\Delta_i^2}{12} \qquad R = 1\ \text{Ом}$$

$$\sigma^2_{кв\ н.шк} = \sum_{i=1}^{N} p_i\frac{\Delta_i^2}{12}$$

$$\sigma^2_{кв\ р.шк} = \left.\sum_{i=1}^{N} p_i\frac{\Delta_i^2}{12}\right|_{\Delta_i=const=\Delta} = \frac{\Delta^2}{12}\sum_{i=1}^{N} p_i = \frac{\Delta^2}{12}$$

Защищенность от шумов квантования при равномерной шкале

$$A_{з\ кв} = 10\lg\frac{P_c}{\sigma^2_{кв}}$$

$$P_c = \frac{U^2}{2} \qquad R = 1\ \text{Ом}$$

$$A_{з\ кв} = 10\lg\frac{U^2 12}{2\Delta^2} \qquad \Delta = \frac{2U_{огр}}{N-1} \approx \frac{2U_{огр}}{2^m}$$

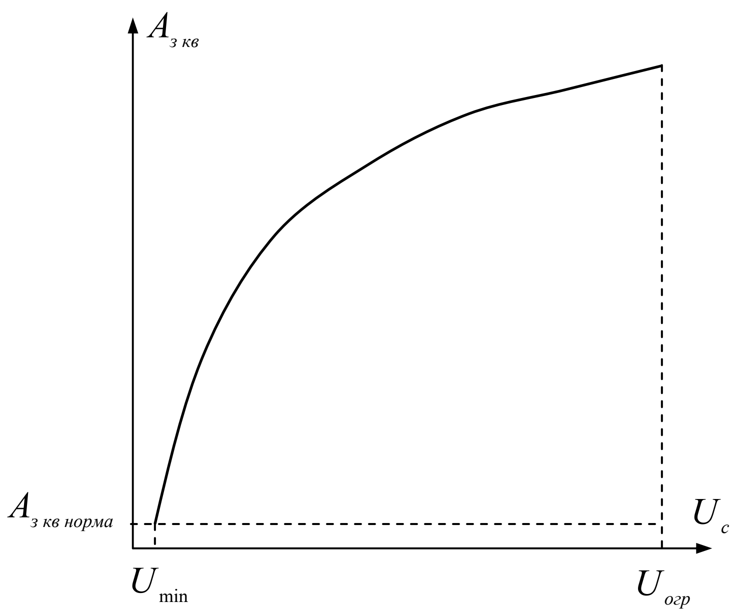
$$A_{з\ кв} = 10\lg\frac{6U^2 2^{2m}}{4U^2_{огр}} = 20\lg\frac{U}{U_{огр}} + 10\lg\frac{3}{2} + 10\lg 2^{2m}$$

$$A_{з\ кв} = 6m + 1{,}8 + 20\lg\frac{U}{U_{огр}}$$

$$\left|U\right| \le U_{огр} \to \frac{U}{U_{огр}} \le 1 \to 20\lg\frac{U}{U_{огр}} \le 0 \quad \to \quad A_{з\ кв}\Big|_{\left|U\right|=U_{огр}} = A_{з\ кв\ \max} = 6m + 1{,}8$$

$$A_{з\ кв\ \min} = A_{з\ кв}\Big|_{U=U_{\min}} \qquad U \to U_{\min} \quad U_{огр} \to U_{\max} \qquad 20\lg\frac{U}{U_{огр}} = 20\lg\frac{U_{\min}}{U_{\max}} = -D$$

$$A_{з\ кв} = 6m + 1{,}8 - D$$



$$m = \frac{A_{з\,кв} + D - 1{,}8}{6} \qquad A_{з\,кв} = 30\,дБ \quad D = 40\,дБ \rightarrow m = 11,.... \rightarrow m = 12\,бит$$

$$W = f_Д m = 8\,кГц \cdot 12\,бит = 96\,кбит/с \rightarrow f_Т = 96\,кГц$$

Алгоритм кодирования (кодер взвешивания)

$U_с \quad m$

$a_1\, a_2\, a_3 \ldots\ldots a_m$

$a_1 - знак\, U_с \; (+) - "0"$

$\qquad\qquad (-) - "1"$

$a_2\, a_3 \ldots\ldots a_m \quad - величина\, |U_с|$

$(m-1)\; эталонов$

$$\left.\begin{array}{l} 2^0\Delta = \Delta \\ 2^1\Delta = 2\Delta \\ 2^2\Delta = 4\Delta \\ \vdots \\ 2^{m-2}\Delta = 2^{m-2}\Delta \end{array}\right\} (m-1)$$

$|U_с| < 2^{m-2}\Delta \rightarrow a_2 = "0" \rightarrow |U_с|_1 = |U_с|$

$|U_с| > 2^{m-2}\Delta \rightarrow a_2 = "1" \rightarrow |U_с|_1 = |U_с| - 2^{m-2}\Delta$

$|U_с|_1 < 2^{m-3}\Delta \rightarrow a_3 = "0" \rightarrow |U_с|_2 = |U_с|_1$

$|U_с|_1 > 2^{m-3}\Delta \rightarrow a_3 = "1" \rightarrow |U_с|_2 = |U_с|_1 - 2^{m-3}\Delta$

$\vdots$

$|U_с|_{m-2} < \Delta \rightarrow a_m = "0"$

$|U_с|_{m-2} > \Delta \rightarrow a_m = "1"$

$$знак\ в\ зависимости\ от\ a_1 \pm \left(a_2 \cdot 2^{m-2}\Delta + a_3 \cdot 2^{m-3}\Delta + \ldots + a_m \cdot \Delta + \frac{\Delta}{2} \right)$$

Пример

$m = 5 \quad \Delta = 1\text{мВ}$

$U_c = -9{,}7\text{мВ} = -9{,}7\Delta$

$-9{,}7\Delta < 0 \quad \rightarrow a_1 = 1$

$4\ \textit{эталона}: \Delta,\ 2\Delta,\ 4\Delta,\ 8\Delta$

$9{,}7\Delta > 8\Delta \rightarrow a_2 = 1 \rightarrow 9{,}7\Delta - 8\Delta = 1{,}7\Delta$

$1{,}7\Delta < 4\Delta \rightarrow a_3 = 0 \rightarrow 1{,}7\Delta$

$1{,}7\Delta < 2\Delta \rightarrow a_4 = 0 \rightarrow 1{,}7\Delta$

$1{,}7\Delta > \Delta \rightarrow a_5 = 1$

11001

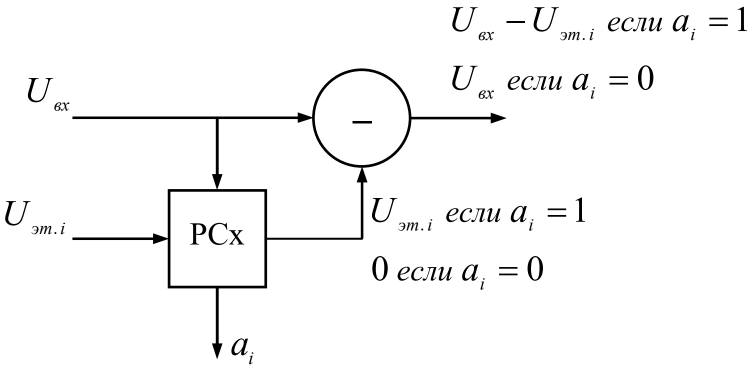
$-(1 \cdot 8\Delta + 0 \cdot 4\Delta + 0 \cdot 2\Delta + 1 \cdot \Delta) = -9\Delta$

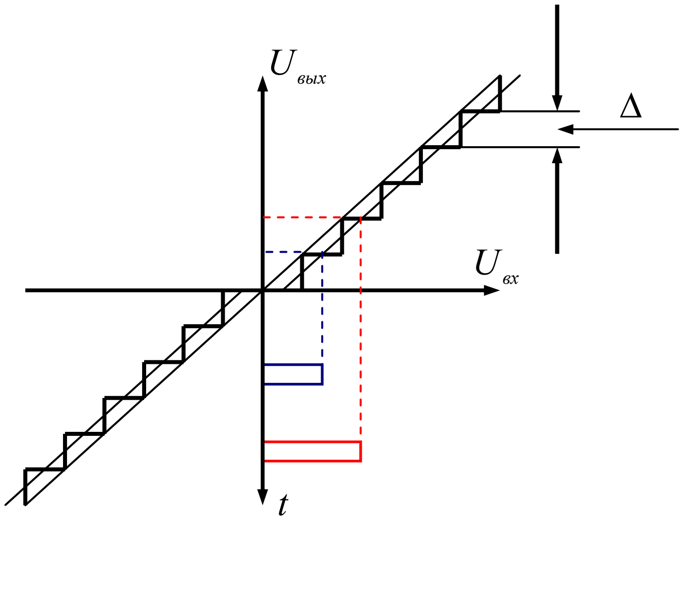
$\varepsilon_{кв} = 0{,}7\Delta > \frac{\Delta}{2}$

коррекция $\quad \frac{\Delta}{2}$

$-\left(1 \cdot 8\Delta + 0 \cdot 4\Delta + 0 \cdot 2\Delta + 1 \cdot \Delta + \frac{\Delta}{2}\right) = -9{,}5\Delta$

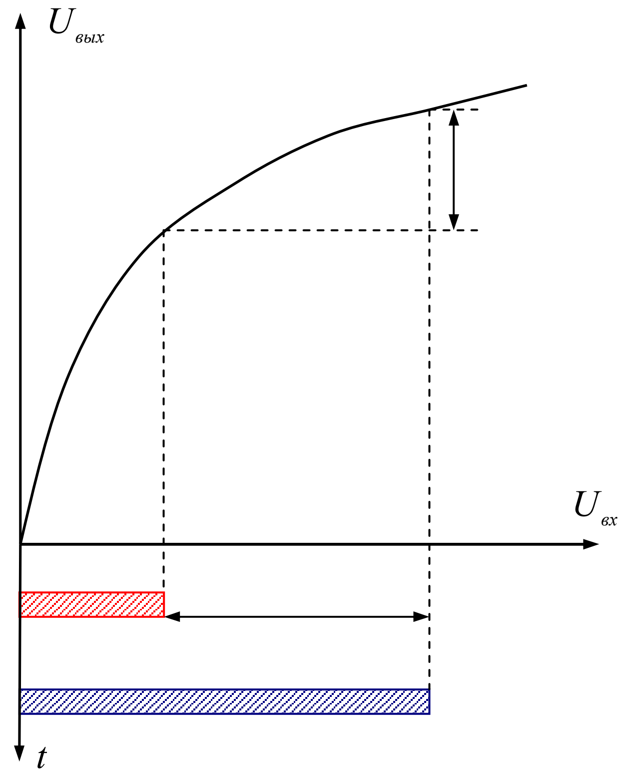
$\varepsilon_{кв} = 0{,}2\Delta < \frac{\Delta}{2}$

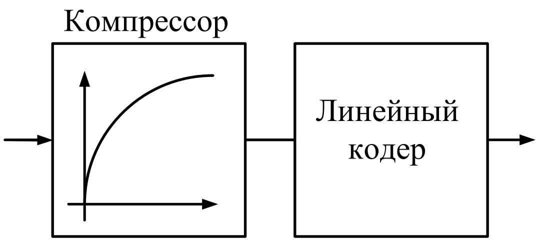


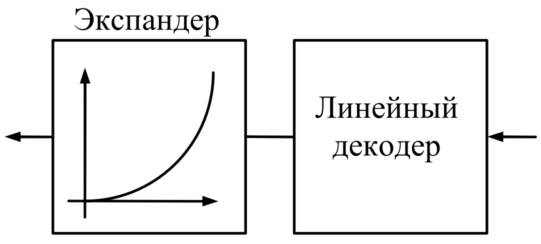


Неравномерная шкала квантования

$$A_{з\ кв} = 6m + 1{,}8 - D$$







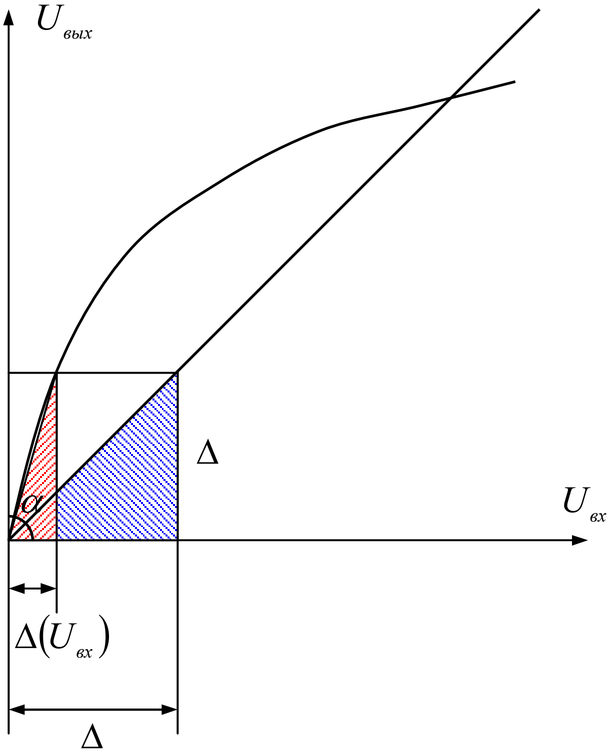
$$m_р = m_н \qquad A_{з\ кв}\Big|_{m=m_р} = A_{з\ кв\ р} = 10\lg\frac{P_с}{\sigma^2_{кв\ р}} = A_{з\ кв\ норма}$$

$$A_{з\ кв}\Big|_{m=m_н} = A_{з\ кв\ н} = 10\lg\frac{P_с}{\sigma^2_{кв\ н}}$$

$$\Delta A_{з\ кв} = A_{з\ кв\ н} - A_{з\ кв\ р} = 10\lg\frac{P_c}{\sigma^2_{кв\ н}} - 10\lg\frac{P_c}{\sigma^2_{кв\ р}} = 10\lg\frac{\sigma^2_{кв\ р}}{\sigma^2_{кв\ н}}$$

$$\sigma^2_{кв\ р} = \frac{\Delta^2}{12} \qquad \sigma^2_{кв\ н} = \sum_{i=1}^{N} p_i \frac{\Delta_i^2}{12} = \frac{\Delta^2(U_{вх})}{12}$$

$$\Delta A_{з\ кв} = 10\lg\frac{\Delta^2}{\Delta^2(U_{вх})} = 20\lg\frac{\Delta}{\Delta(U_{вх})}$$



$$\frac{\Delta}{\Delta(U_{вх})} = tg\alpha = \frac{dU_{вых}}{dU_{вх}} \qquad \Delta A_{з\ кв} = 20\lg\frac{dU_{вых}}{dU_{вх}}$$

$$A_{з\ кв\ н} = 10\lg\frac{U^2_{вх}12}{2\Delta^2(U_{вх})} = const \quad \rightarrow \Delta(U_{вх}) = kU_{вх}$$

$$\frac{\Delta}{kU_{вх}} = \frac{dU_{вых}}{dU_{вх}}$$

$$U_{вх} \rightarrow \frac{U_{вх}}{U_{огр}} = x \qquad U_{вых} \rightarrow \frac{U_{вых}}{U_{огр}} = y$$

$$\frac{dy}{dx} = \frac{\Delta}{kU_{огр}} \frac{1}{x} \qquad \frac{dy}{dx} = B\frac{1}{x}$$

$$y = B\ln(x) + C$$

$$U_{вх} = U_{огр} \to x = 1 \qquad U_{вых} = U_{огр} \to y = 1$$
$$U_{вх} = 0 \to x = 0 \qquad U_{вых} = 0 \to y = 0$$

$$(1\,,1) \qquad 1 = B\ln(1) + C \to C = 1$$
$$(0\,,0) \qquad 0 \neq B\ln(0) + 1$$

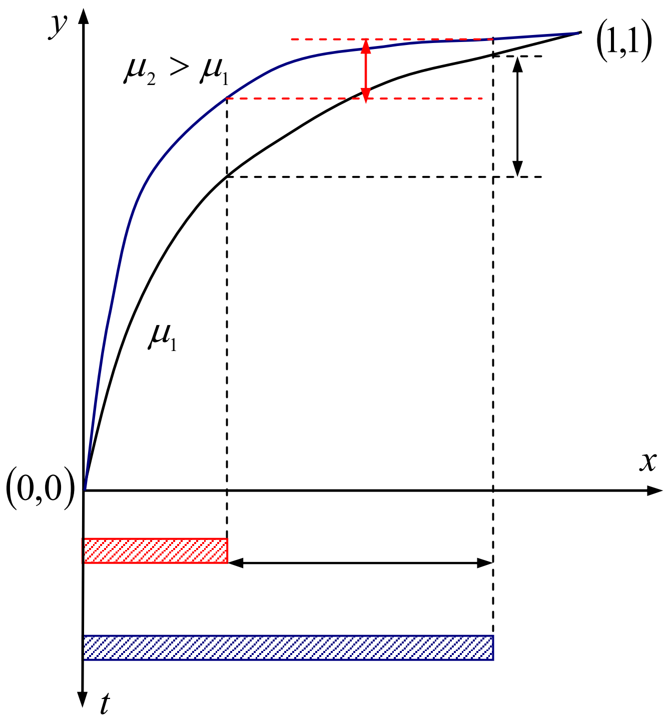
$$y = B\ln(1 + \mu x) + C$$

$$(0\,,0) \qquad 0 = B\ln(1) + C \to C = 0$$
$$(1\,,1) \qquad 1 = B\ln(1 + \mu) \to B = \frac{1}{\ln(1 + \mu)}$$

$$y = \frac{\ln(1 + \mu x)}{\ln(1 + \mu)},\ x \geq 0$$

$$y = \frac{\ln(1 + \mu|x|)}{\ln(1 + \mu)} sign(x),\ -1 \leq x \leq 1$$

$$\mu = 100 \qquad \mu = 255$$



$$y=\begin{cases}\dfrac{Ax}{1+\ln A}, & 0\le |x|\le \dfrac{1}{A},\\ \dfrac{1+\ln\left(A|x|\right)}{1+\ln A}\operatorname{sign}(x), & \dfrac{1}{A}\le |x|\le 1,\end{cases}$$

$$u_{\text{вых}}(u_{\text{вх}})=\begin{cases}\dfrac{Au_{\text{вх}}/U_{\text{огр}}}{1+\ln A}, & 0\le \dfrac{|u_{\text{вх}}|}{U_{\text{огр}}}\le \dfrac{1}{A},\\ \dfrac{1+\ln\left(A|u_{\text{вх}}|/U_{\text{огр}}\right)}{1+\ln A}\operatorname{sign}(u_{\text{вх}}), & \dfrac{1}{A}\le \dfrac{|u_{\text{вх}}|}{U_{\text{огр}}}\le 1,\end{cases}$$

$$A=87{,}6$$

$$\Delta A_{з\,кв}=20\lg\frac{dU_{вых}}{dU_{вх}}=20\lg\frac{dy}{dx}=20\lg\left.\frac{A}{1+\ln A}\right|_{A=87,6}\approx 24 дБ$$

$$m_р=12\,бит\rightarrow m_н=8\,бит$$

$$W=f_Д m=8\,кГц\cdot 8\,бит=64\,кбит/с$$